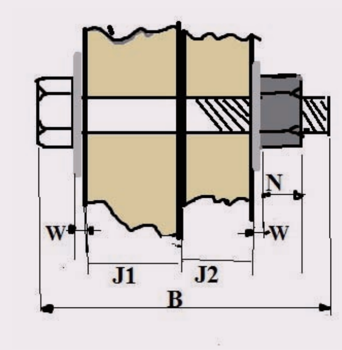
The Why and How of **Spring Rates**

by Thomas Doppke



Many articles mention and use the term "spring rate" or "joint spring rate" but evidently few understand just what it is and why it is important. To understand this concept, we must explore some basic factors of bolting technology. While an excellent tool for joint analysis, there are factors that can and will impact the calculations that need to be considered. First, what is a spring rate?

Why are things bolted together? Obviously it is to keep them together. To accomplish this the joint must hold together more tightly than any of the outside forces that are trying to pry it apart (loads, cyclic stresses, gravity, vibration, etc.). Question 2- how tight must they be bolted together to prevent this from occurring? This is the crux of fastening engineering!! Some of the things that affect the tightening are; the strength of the bolt, the size, the length of the bolt, the entire joint thickness (grip range), the material being joined (soft, yielding, rough surfaced, etc.), the way the tightening forces are applied, environmental conditions, the service life conditions (the way that the joint lives-static, dynamic, cyclic impacts, etc.), the expected life of the joint (nothing holds together forever!), and probably a few others.

As our illustration shows, a joint is made up of several parts. When loaded (tightened) the bolt is under tensile loading and the other parts of the joint are under compression. The forces are equal to each other or they would be moving! Each part flexes slightly and acts somewhat like a spring (hence the name spring rate). The bolt is a tension spring, being stretched ever so slightly by the tightening of the joint components. The amount of spring rate is an important factor in determining the portion of the final assembled loading that the fastener (here the bolt) can carry and in determining the ability of it to compensate for any yielding that may occur. If the bolt is subjected to tension forces (over torqueing or perhaps service loads) that exceed the tensile strength of the bolt, failure will occur. Conversely, if service loads decrease the compressive forces (permanently or cyclically) the preload can be reduced to as little as zero, instituting conditions of fatigue for the joint.

Rather than wait for field failures to arrive, usually with your Discharge Notice from the boss, or spend much money on exemplar testing and prototype builds (another way to get front office attention) some rough calculations can be made which will give you a hint as to which way to go. Remember that we consider the entire joint to be similar to a spring. Each part contributes to some part of the loading, the bolt in tension, the other parts mostly in compression. Other factors may apply also, embedment, cold flow (creep), fretting, and vibrational wear all play a part -more on these later.

It is impractical to measure actual fastener spring rate for every application. Besides being very costly the difficulties in obtaining individual component values make the results suspect at best. The following method is not 100% accurate but is satisfactory for most joints.

Since the joint members are to be thought of as "springs" their rates may be derived from Hooke's Law that strain is proportional to stress:

$$\Delta L = \frac{F \times L}{E \times A}$$

Where: F = Applied force

L = Length of cross section

E = Modulus of Elasticity

A = Cross section area

ΔL = Change in length of cross section

Then the spring rate can be defined as: $K = \dfrac{F}{\Delta L}$

Where: F = Applied force

K = Spring Constant

ΔL = Change in length of cross section

Solving by substitution we arrive at: $K = \dfrac{A \times E}{L}$

With this equation in hand, we can determine the spring rate of the bolt. We determine the spring rate of each element of the bolt and add them in series. The Bolt Total Spring Rate (Kb) is:

$$Kb = \frac{1 \quad + \quad 1 \quad + \quad 1}{Kh \qquad Ks \qquad Kt}$$

Where Kh, Ks, and Kt are the spring rates of the head, shank and threads respectively. Using our equation from above we can obtain the bolt head's deflection (Kb) from Hooke's Law. Instead of deflection testing the bolt's head under load, we approximate it assuming that the nominal shank diameter (D) is used for area and the length (L) as one half of the bolt head thickness. Trust me, this works but the mathematics are too much a headache to show here. This gives an area that is almost similar to that obtained by actual testing. Expressed as:

$$Kh = \frac{[\,\Pi\, D^2 \text{ x } E\,]}{L}$$

Using nominal body diameter (D) and the shank body length (L) the shank spring rate (Ks) can be approximated with this same equation.

The bolt thread spring rate (Kt) uses the same equation but uses L as the thread length plus one half of the mating internal thread height (nut height) and A for the thread tensile stress area. Again the mathematical explanation for this is best left for another article. Trust me, it works.

This is expressed as:

$$Kt = \frac{\text{Thread tensile stress area x } E}{L}$$

Confusing? Not so much. Here is an example. Using a standard M12 x 1.75 x 100 bolt with a standard nut we have this data;

Bolt head height----------------------11.8mm

unthreaded shank--------------70mm

tensile stress area (thread)----84.3mm^2

bolt length-----------------------100mm

Modulus of Elasticity----------.207MPa (from charts)

Nut height-----------------------------11.8mm

Joint thickness-------------------------84mm

Using the bolt spring rate calculations:

Kh = 396799 N/mm

Ks = 334445 N/mm

Kt = 876889 N/M

Into the Kb formula: $$Kb = \frac{1}{\frac{1}{Kh} + \frac{1}{Ks} + \frac{1}{Kt}}$$

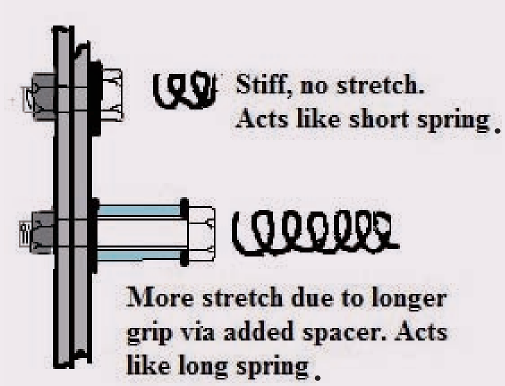
We find the bolt spring rate to be 228182 N/mm.

With the bolt spring rate now in hand we can determine the joint spring rate. Or rather make an approximation. The most accurate way is to run an exemplar test but since variations in bolts, strengths, configuration of parts, etc., make any determination of an accurate cross section almost impossible it is possible to make an approximation. Logically the joint has an outside diameter equal to or larger than the minimum bolt head bearing face diameter. Our example bolt has, from the data, a bearing face diameter of 22.5 mm minimum and with a fillet radius of 0.5mm. Using Hooke's Law again and this data we find that the joint has a spring rate of 677400N/mm. By comparison, almost three time stiffer than the fastener in a solid joint. Excellent for a joint expected to live a long and successful life.

The bolt spring rates in actual joints will usually be found to be lower than the calculated values. Among the factors that may alter the value are changes in grip length. The example was for a solid joint. Some joints may show a decrease in their grip range, due to such factors as included gasketed members and/or softer materials (plastics, soft metal), and air gaps due to non-surface to surface contact. The change in the value as generally to a lower value. Also dimensional changes to the bolt (grooves, reduced shank diameter, thread formation and length of thread) impact the final numbers.

Short bolt lengths increase stiffness while longer bolts act like longer springs and yield lower spring rates. Often the use of a longer bolt with a spacer is seen in applications where potential (or actual) loosening occurs. A short bolt may not be capable of being tightened enough to achieve the desired preload to secure the joint. It will not stretch enough! The use of a spacer increases the joint grip range by allowing a longer fastener to be used. This is to increase the stretch of the bolt, allowing for the application of a higher torque (preload). (A longer spring stretches more than a short one!).

A common application is the steering gear to frame attachment on heavy vehicles. The service loads and cyclic impacts may exceed the bolt's tension and momentary separation occurs (load goes to zero). Cyclic loading from zero to preload and back is the basis for fatigue failure. If the cyclic swings exceed



the joint strength on the high end, the bolt may go into bolt yield and permanent elongation may occur. This permanent additional lengthening of the bolt causes a reduction in cross sectional area. The reduced cross sectional area is less capable of carrying the load and cycle after cycle reduces the bolt strength until a failure caused by the hysteresis occurs.

As mentioned before, another decrease factor in spring rate is the embedding and settling of the fastener when installed. Softer materials will settle and tend to cold flow away from the tightened area after time. While there is no mathematic formula to compute the actual amount that affects the spring rate we can make a rough guess as stated below. Assuming the settling (loss of load) occurs during the first assembly of the joint and no further settling occurs in any further installations a quick calculation is:

$$Ls = El \frac{(SRa - 1)}{SRo}$$

Ls = Load loss from settling

El = External load applied to fastener

SRa = Spring rate after assembled

SRo = Spring rate original (before settling)

Data from actual tests of various sizes and conditions show that the settled spring rates were in the area of 125-150% of the pre-assembly rates. These same tests also showed that the calculated spring rates, when compared to actual values, agreed to within 6-7% difference.

Among the other factors that affect spring rates are the vibrational effects, both large and small. Vibration is the most dangerous factor in shortening a joint's life. We have figured out our joint spring rate; everything is good, joint is solid, it will last forever (or until warranty is over). What can go wrong now? The loss of preload involves very little actual loss at all. A fastener can be taken from full preload to zero in as little as ½ turn (that's half a thread pitch!). This is the basis of the 'Turn of the Nut" method of assembly. It is actually more accurate than many modern tools, i.e., impact guns and torque bar control tools. The amount to exceed the yield strength on most steel fasteners is above the proof load (commonly given as either a value or a stretch of approximately so many millimeters). Vibration, especially cyclic vibration, from zero to preload and back cycles, or the greater than yield strength and back cycles, until something else gives, reduces the preload. If great reduction occurs, we have failure.

Fretting is the minute vibrational forces acting on a joint, and never exceeds the yield strength at any point. Like the actions of sandpaper, the small vibrations "sand" away at asperities and bumps and ridges until the contact surfaces are 'smoother'. This reduces the grip range also. As said above, only a small amount of reduction can lead to a great reduction in preload, which leads to …!

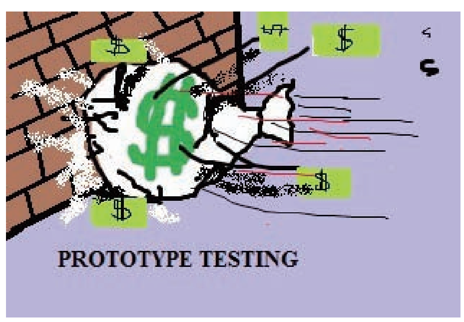
The fastener is always, except for rare and unusual conditions in exotic applications, fastened by torqueing either the head or nut. Automatic tooling runs quickly and the impact at tightened level is sudden. Tests have shown that the tension value obtained by torqueing is slightly different than that value computed for a true tension via stretch. While the actual amount varies greatly with material considerations, hardness, joint makeup and other factors the approximate average of immediate tensile loss is 2-25% and an overall loss after a period of several days of another 5-9%. As always, when in doubt, test!

While torque is the chosen method of attaining the requisite tensile preload in the joint, values arrived at by torsional means are estimated to be some 15% lower than expected. The loss is due to torsional 'wind up' loss which deducts from the anticipated value that a torque is expected to produce in tensile. Bolt stretch is the most accurate way to measure actual tension in a fastened bolt. Alas, it is extremely difficult to measure without graphing stretch vs. load first and reproducing it in the application.

Various tools also vary the output of the fastened joint. Some tools vary so much from their set points that the actual value is a guess (impact guns are +/- 25% at best). Torque control bar tools are nearly +/- 15% and the best on the market are complex systems with expensive electronics which are a +/-1% of set.

Spring rate calculations offer an inexpensive way to check out the design. Prototype builds now approach US$250,000 for each variation that is durability tested, run into a wall, or road tested. And that doesn't include the time lost in fixing the found problems. Every dollar saved is a competitive advantage in the market place.

Joint spring rate is a useful tool in determining how a joint will function under varying load conditions. What seems a workable joint can be found to be possibly marginal when the conditions of service loads and other variables are considered.



Rather than waiting for a field failure or expending expensive laboratory and prototype time and resources, a calculation can be made to approximate the durability of the joint. The spring rate calculation is a quick and easy way to look at overall joint stress. It is important to consider the other factors mentioned above that may influence the obtained value used in the calculations. Joint spring rate is a fast way to review joint analysis without getting into lengthy computer modeling and stress analysis programs. In cases where the calculations indicate a possible problem a more detailed study of that particular area is in order.